CPE201
Digital Design

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Class 2: Number Systems

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Corrections

- No partial credit for either submission of assignments

Announcements

- Discord server for this course – see Syllabus
- Labs start this week
- First homework due next Wednesday
- When HW is posted
- Makeup and resubmission lab
  - Thursdays 1-5pm

To the Experts here

- “What were we put here to do if not help each other through?” -Mark Horstman

Outline

- Number representations (number systems)
- Conversions+

Decimal

- The system you are used to
  - There are 10 digits (0 [the first] to 9 [the tenth])
- When you use all the digits, a new digit goes in the next position up

$$42 = (4 \times 10) + (2 \times 1)$$

- The new position has added weight

Decimal

- Each digit to the left has increasing weight

$$10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0$$

- Weights continue to work after a decimal point

$$10^2\ 10^1\ 10^0\ .\ 10^{-1}\ 10^{-2}\ 10^{-3}$$

Example

851.96

$(8 \times 10^{2}) + (5 \times 10^{1}) + (1 \times 10^{0}) + (9 \times 10^{-1}) + (6 \times 10^{-2})$

$(8 \times 100) + (5 \times 10) + (1 \times 1) + (9 \times 0.1) + (6 \times 0.01)$

800 + 50 + 1 + 0.9 + 0.06

- The weight on the 6 is 0.01 (or 1/100)

Decimal

Power of 10	Decimal
10^{0}	1
10^{1}	10
10^{2}	100
10^{3}	Thousand or Kilo
10^{6}	Million or Mega
10^{9}	Billion or Giga
10^{12}	Trillion or Tera

Power of 10	Decimal
10^{-1}	Tenth
10^{-2}	Hundredth
10^{-3}	Thousandth or Milli
10^{-6}	Micro
10^{-9}	Nano
10^{-12}	Pico

Binary

- Same a decimal, but there are only 2 digits (bits)
- 0 then 1 then 10 then 11 then 100
- Weight increases with each digit to the left
  - The base is different (2 instead of 10)

$$100 = (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$

$$= (1 \times 4) + (0 \times 2) + (0 \times 1)$$

$$= 4$$

MSB and LSB

- Left most bit = Most significant bit
- Rightmost bit = Least significant bit

1010

MSB LSB

Binary

- Weights can continue after binary point
  - We usually don't use it

Binary	Decimal
2^{-1}	1/2 or 0.5
2^{-2}	1/4 or 0.25
2^{-3}	1/8 or 0.125

Binary and Decimal

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Decimal	Binary
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Powers of 2

Power of 2	Decimal
2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64

Power of 2	Decimal
2^7	128
2^8	256
2^9	512
2^{10}	1,024 = 1k
2^{20}	1,048,576 = 1M
2^{30}	1,073,741,824 = 1G
2^{40}	1T

Bits needed

- How many bits to represent a decimal number?
- Largest decimal number = $2^n - 1$
  - Need to include zero
  - How many bits is n

$$2^2 - 1 = 3$$
$$2^3 - 1 = 7$$
$$2^4 - 1 = 15$$

Octal

- Base 8 number system
  - Not used a lot these days
  - chmod command

Decimal	Octal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10
9	11

Hexadecimal

- Base 16 Number System
- Need extra digits that aren't in decimal system

Decimal	Hex
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

Decimal	Hex
9	9
10	A
11	B
12	C
13	D
14	E
15	F
16	10
17	11

Converions

- Binary, Octal, Hex, and Decimal

$$101100111110_2 = 5476_8 = B3E_{16} = 2878_{10}$$

also

0b101100111110=0o5476=0xB3E=2878

Binary to Octal/Hex and vice versa

- Easiest
- Hex is a grouping of 4 bits ($2^4 = 16$)
- Octal is a grouping of 3 bits ($2^3 = 8$)

Example

$101100111110_2 = 101\ \ 100\ \ 111\ \ 110$

$= 5 \quad 4 \quad 7 \quad 6$

$= 5476_8$

$101100111110_2 = 1011\ \ 0011\ \ 1110$

$= B \quad 3 \quad E$

$= B3E_{16}$

Example

$A49_{16}$ = A 4 9

= 1010 0100 1001

= 101001001001_2

713_8 = 7 1 3

= 111 001 011

= 111001011_2

Binary to Decimal

- Use your powers of 2 and add

11001010_2 -7 bits

$2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0$

1 1 0 0 1 0 1 0

$2^7+2^6+2^3+2^1$

128+64+8+2 = 202

Decimal to Binary

- Sum of Weights Method
  - Use largest weight that is smaller than you #, then subtract and repeat until zero is left

102 next smallest weight is $64=2^6$

102-64=38 next smallest weight is $32=2^5$

38-32=6 next smallest weight is $4=2^2$

6-4=2 next smallest weight is $2=2^1$

2-2=0 DONE

$102=2^6 2^5 2^2 2^1=1100110_2$

Decimal to Binary

- Divide by 2 Method
  - Divide number by 2 repeatedly (whole #s and remainders until zero is left)

102/2=51 even # to start, so no remainder = 0

51/2=25 odd # to start, so remainder = 1

25/2=12 odd # to start, so remainder = 1

12/2=6 even # to start, so no remainder = 0

6/2=3 even # to start, so no remainder = 0

3/2=1 odd # to start, so remainder = 1

1/2=0 odd # to start, so remainder = 1 DONE

Put the numbers together backwards (first division was LSB, last one was MSB)

$102 = 1100110_2$

Decimal to Octal/Hex and vice versa

- Decimal -> Binary -> Octal/Hex
- Octal/Hex -> Binary -> Decimal
- You already know all of these steps

Reading

- This lecture
  - Sections 2.1-2.3, 2.8-2.9
- Next lecture
  - Sections 2.4-2.7