CPE201
Digital Design

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Class 14: Adders

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Outline

- Half Adder
- Full Adder
- Parallel Adders
- Ripple vs Look Ahead Carry

Combinational Logic Applications

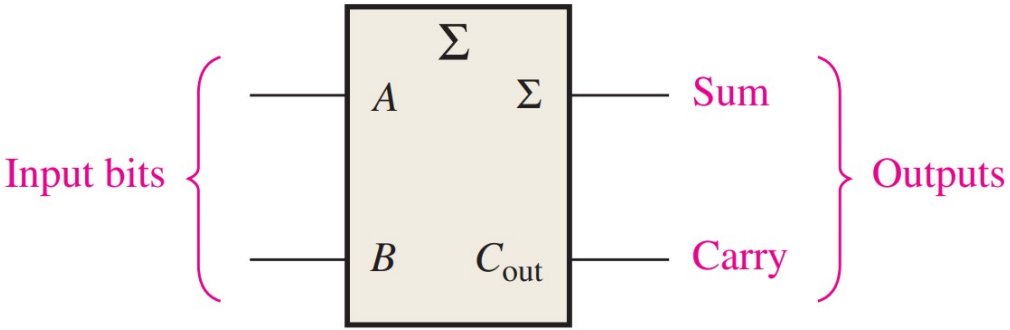
- You now know the basics
  - Let’s make cool stuff!
  - Adders
  - Comparators
  - Decoders/Encoders
  - Mux/Demux
  - And more!

Half Adder

- Recall binary addition
  - 0 + 0 = 0
  - 0 + 1 = 1
  - 1 + 0 = 1
  - 1 + 1 = 10
- A circuit that does this is a half adder

Half Adder

- 2 inputs, 2 outputs
  - Addition of 2 bits

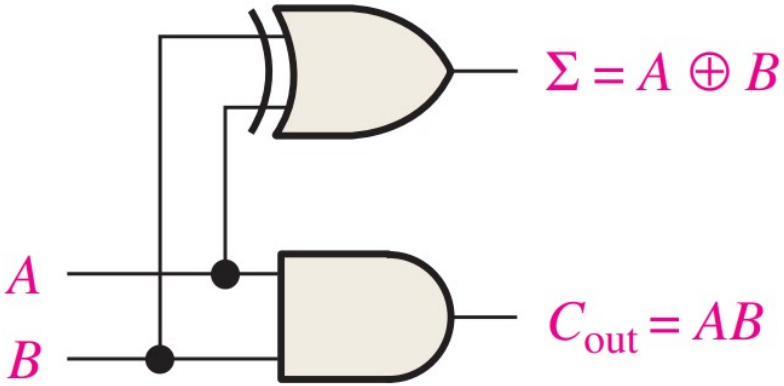


A	B	C_{out}	Σ
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Σ = sum
C_{out} = output carry
A and B = input variables (operands)

Half Adder

- $C_{out} = AB$
- $\Sigma = A \oplus B$



A	B	C_{out}	Σ
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

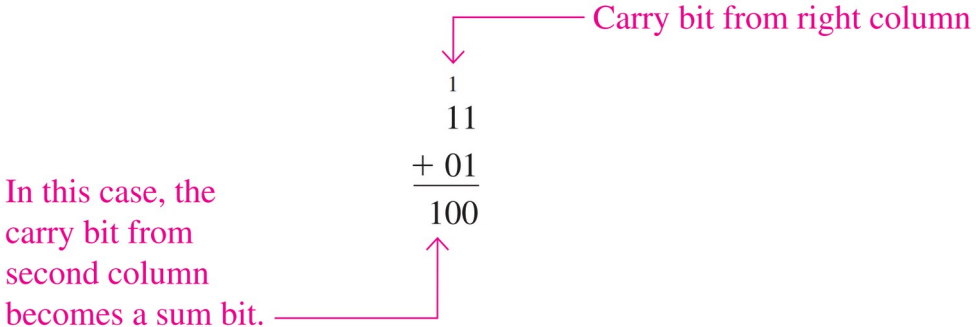
Σ = sum

C_{out} = output carry

A and B = input variables (operands)

Half Adder+

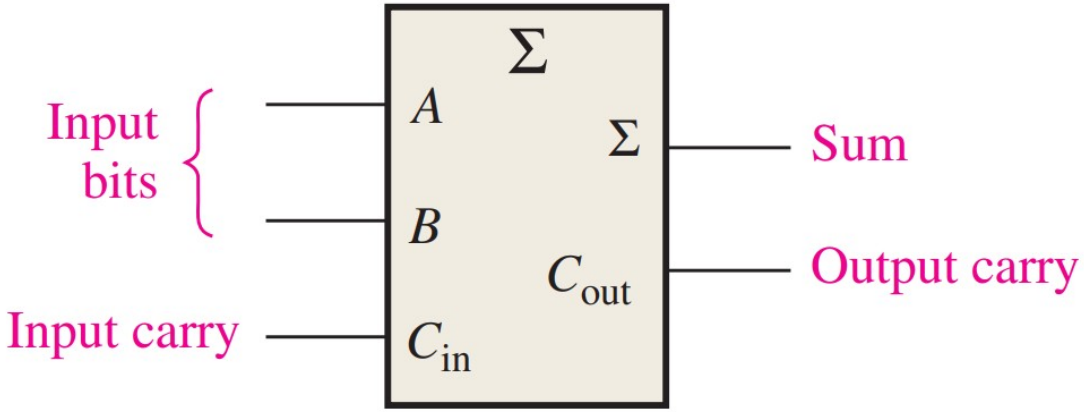
- Can add two input bits together
  - Not complete for full binary addition



- Needs input carry AND output carry

Full Adder

- 3 inputs, 2 outputs
  - Addition of 3 bits



A	B	C_{in}	C_{out}	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

C_{in} = input carry, sometimes designated as CI
C_{out} = output carry, sometimes designated as CO
Σ = sum
A and B = input variables (operands)

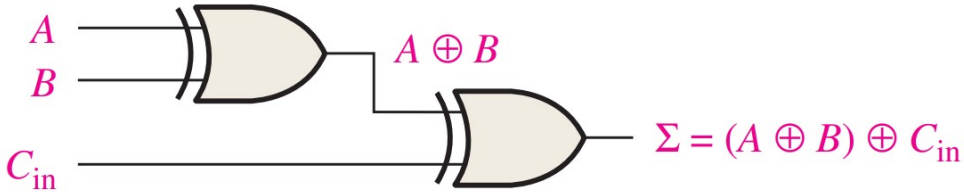
Full Adder

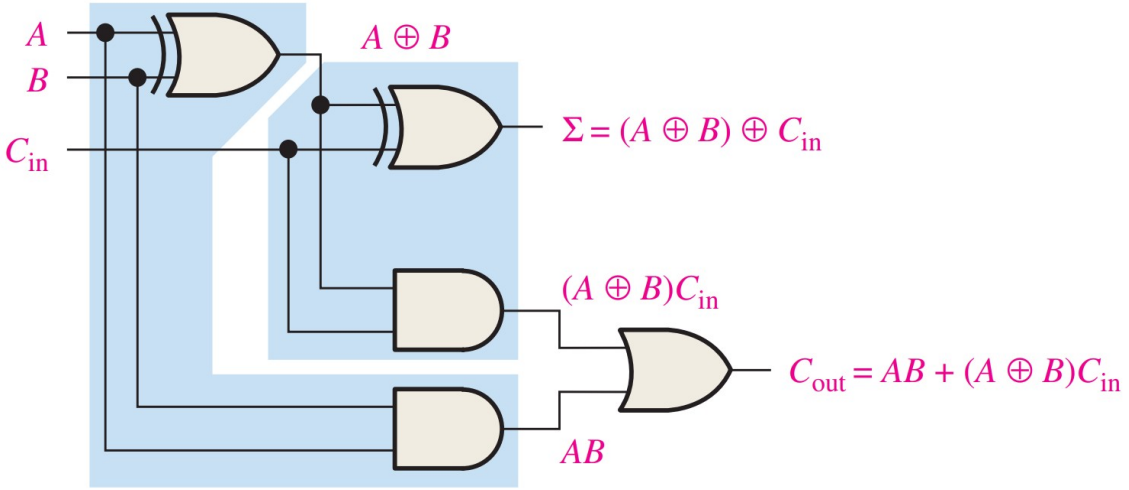
- $C_{out} = AB + (A\ B)C_{in}$
- $\Sigma = (A\ B)\ \ C_{in}$
- Not as obvious

A	B	C_{in}	C_{out}	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

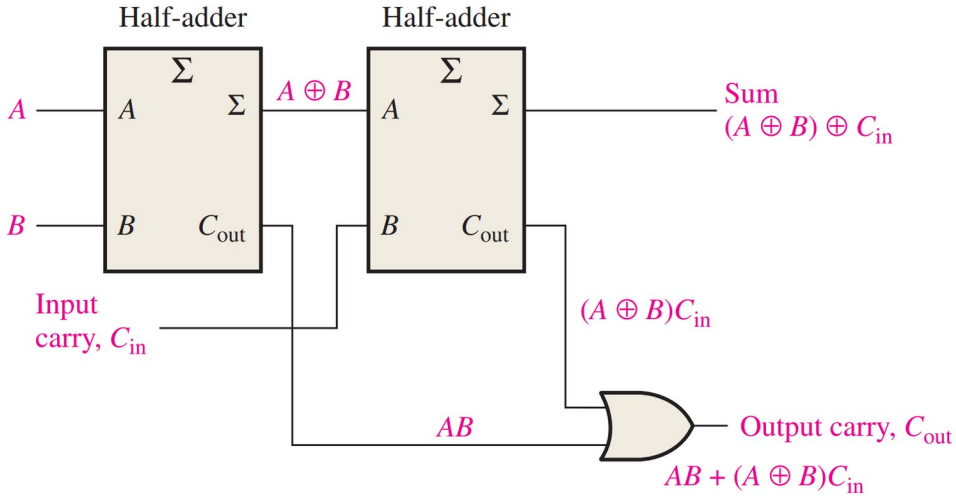
C_{in} = input carry, sometimes designated as *CI*
C_{out} = output carry, sometimes designated as *CO*
Σ = sum
A and B = input variables (operands)

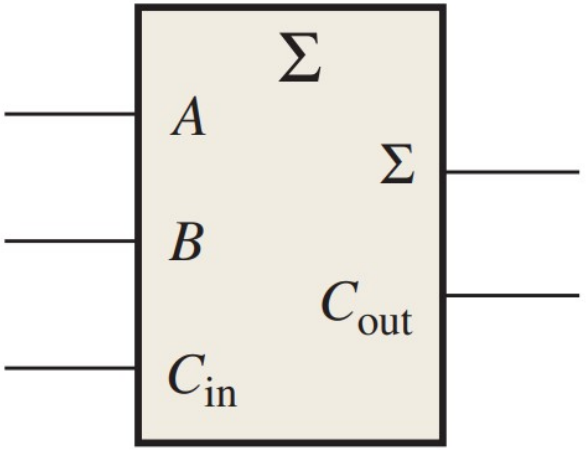
Full Adder


A
B
C_{in}
$A \oplus B$
$\Sigma = (A \oplus B) \oplus C_{in}$


A
B
C_{in}
$A \oplus B$
$\Sigma = (A \oplus B) \oplus C_{in}$
$(A \oplus B)C_{in}$
$C_{out} = AB + (A \oplus B)C_{in}$
AB

Full Adder


Half-adder
Half-adder
A
B
Σ
A
B
Σ
C_{out}
Σ
A
B
Σ
C_{out}
$A \oplus B$
Sum
$(A \oplus B) \oplus C_{in}$
Input
carry, C_{in}
$(A \oplus B)C_{in}$
AB
Output carry, C_{out}
$AB + (A \oplus B)C_{in}$


Σ
A
B
C_{in}
Σ
C_{out}

Full Adder+

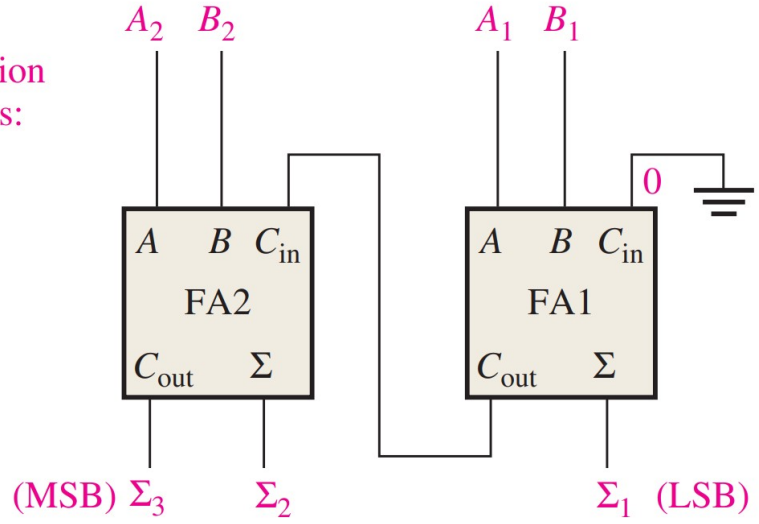
- Can add three input bits together
  - Complete for full binary addition
  - Let’s do that!

Parallel Binary Adders

- Two or more full adders cascaded

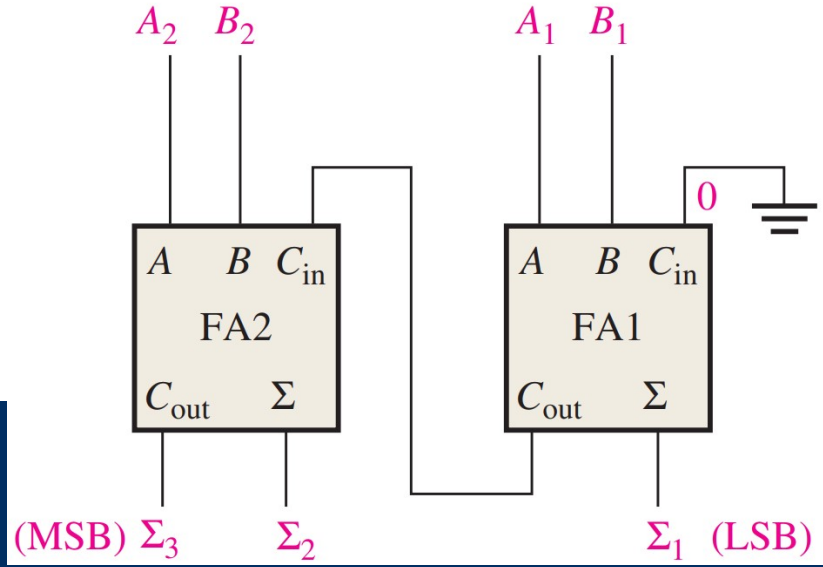
General format, addition of two 2-bit numbers:

$$\begin{array}{r} A_2A_1 \\ +\ B_2B_1 \\ \hline \Sigma_3\Sigma_2\Sigma_1 \end{array}$$



Example

- For 11 + 01 = 100
- A_1=1 A_2=1 B_1=1 B_2=0



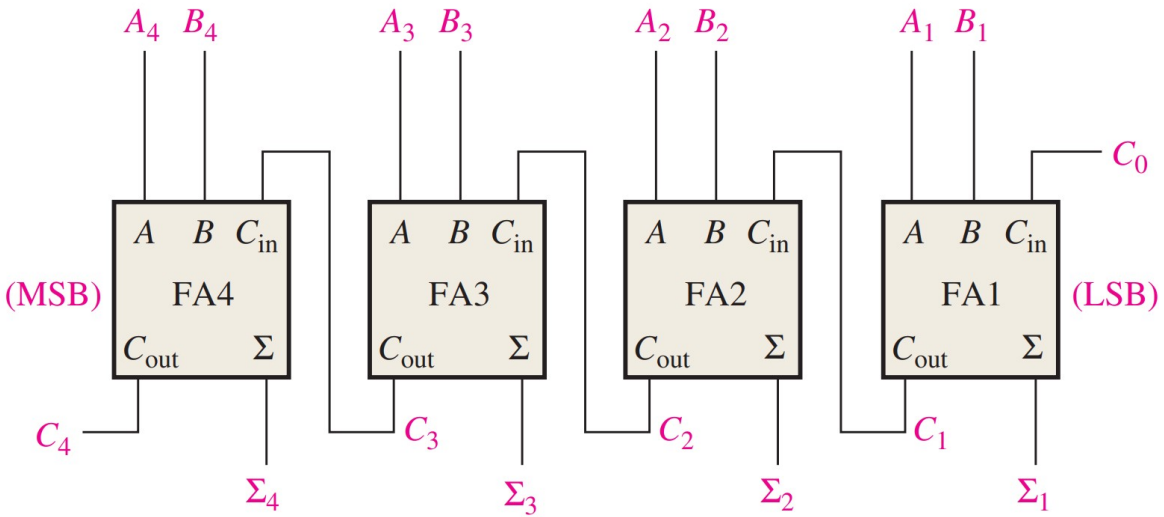
$$\begin{array}{r} A_2A_1 \\ +\ B_2B_1 \\ \hline \Sigma_3\Sigma_2\Sigma_1 \end{array}$$

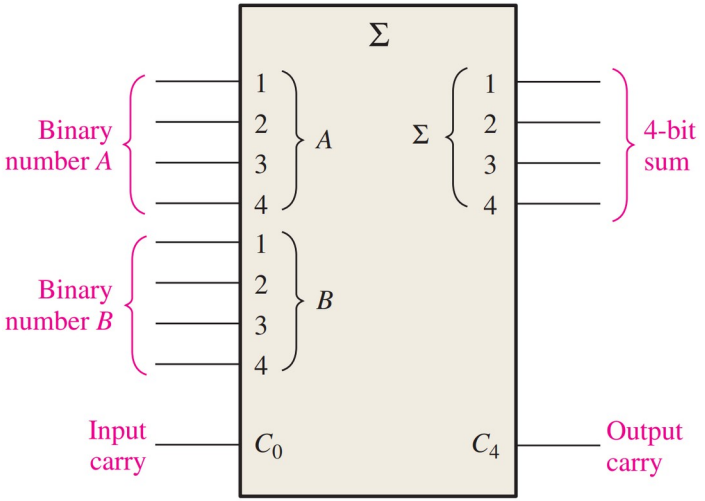
A	B	C_{in}	C_{out}	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

C_{in} = input carry, sometimes designated as *CI*
C_{out} = output carry, sometimes designated as *CO*
Σ = sum
A and B = input variables (operands)

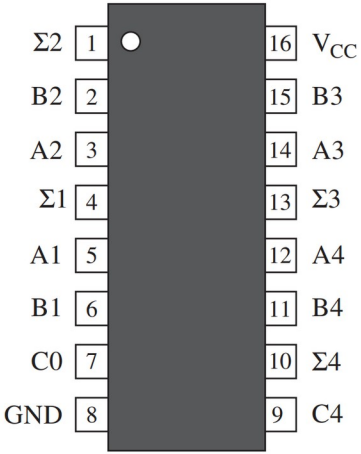
4-Bit Parallel Adder

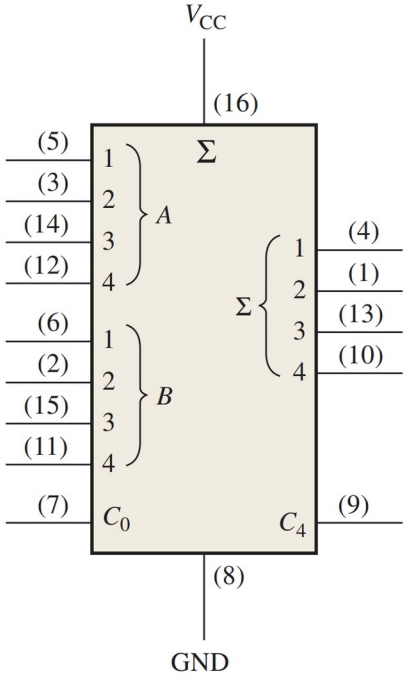
- You can buy those (74xx283)





Nibble Adder

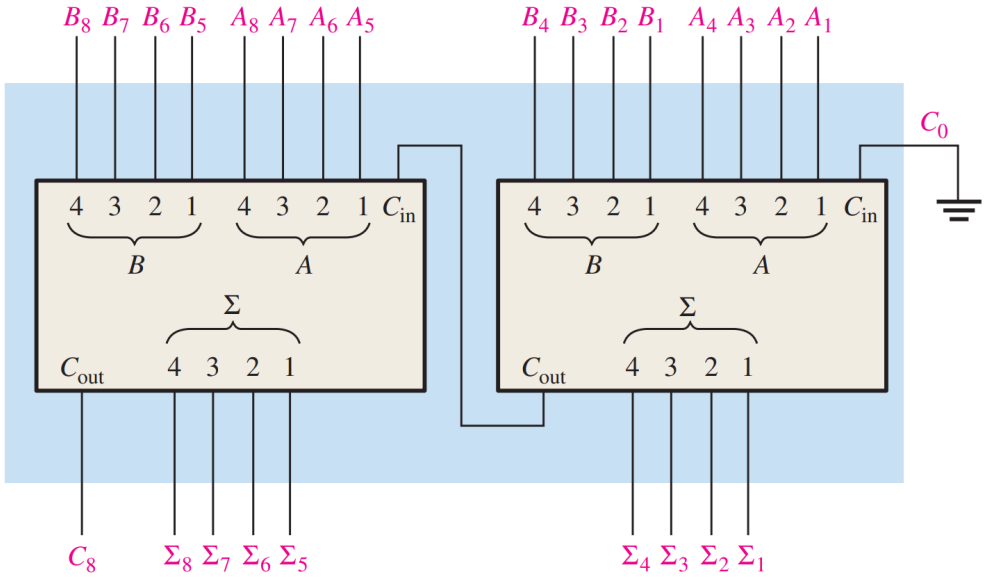




C_{n-1}	A_n	B_n	Σ_n	C_n
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

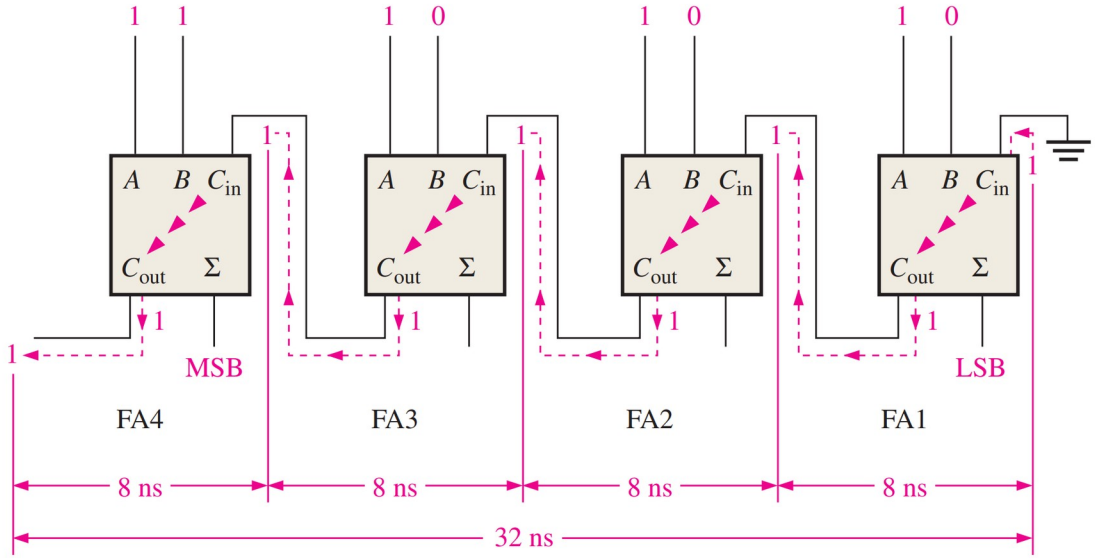
Byte Adder

- More Cascading



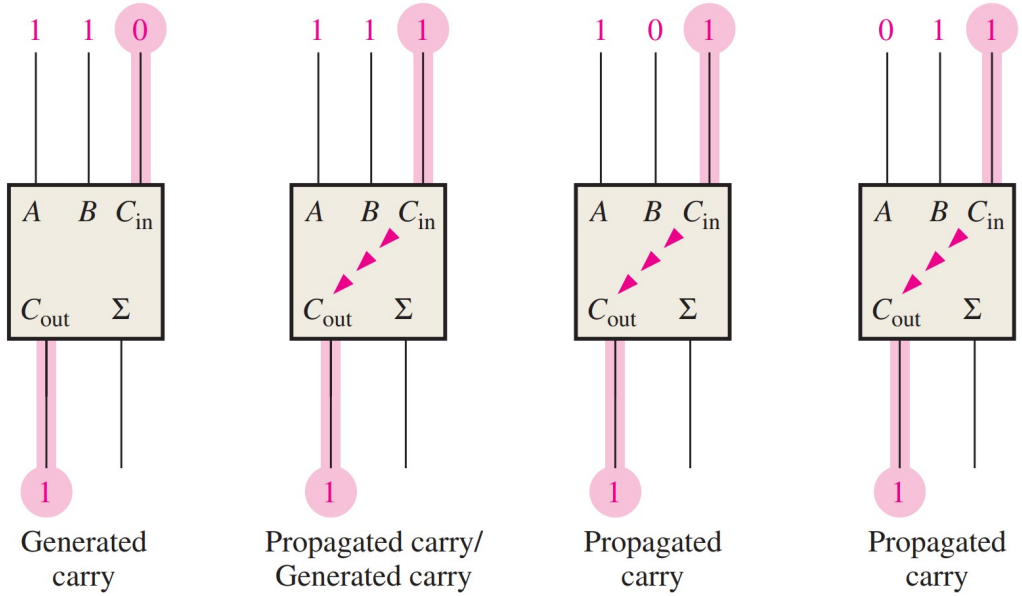
Problem = Ripple Carry Adder

- Carry bits are slow
- Total time = (3n)τ (time for 1 bit of adding)
- Mult/Division = Add/Subtr
- Faster addition makes all others faster



Generated and Propagated Carry

- $C_g = AB$
- $C_p = A + B$ book
- $C_p = A \ B$ web
- $\Sigma = C_p \ C_{in}$
- $C_{out} = C_g + C_p \ C_{in}$
- $C_{out} = AB + (A \ B)C_{in}$ (Full Adder)



Carry Look-Ahead Adder

- All inputs are available at the same time
  - Create all C_gs and C_ps at the same time
- $C_{g,i} = A_i B_i$
- $C_{p,i} = A_i \oplus B_i$
- Same as a Half Adder
- i = 0…3

- Half Adder:
- $C_{out} = AB$
- $\Sigma = A \oplus B$

Carry Look-Ahead Adder

- Create all Carry bits
- $C_{(i+1)} = C_{g,i} + C_{p,i} C_i$
- $C_0 = C_{in}$
- $C_4 = C_{out}$

Carry Look-Ahead Adder

- Last, create all sum bits
- $\Sigma_i = C_{p,i}\ C_i$

Carry Look-Ahead Adder

- Half Adder:
- $C_{out} = AB$
- $\Sigma = A \oplus B$

Carry Look-Ahead Adder

- 3 Stages of Logic with no ripple
- Total time = 4τ (time for 1 bit of adding)
- Ripple carry takes 9τ for 4 bits
- Tradeoff of more complex circuitry (w/ more gates)

Reading

- This lecture
  - Sections 6.1-6.3
- Next lecture
  - Sections 6.4-6.6