# CPE201
# Digital Design

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Class 8: DeMorgan’s and Boolean Simplification

University of Nevada, Reno

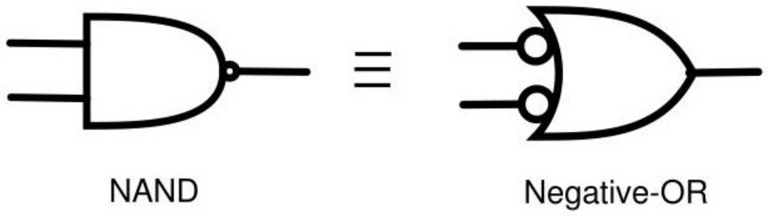
# Mastery

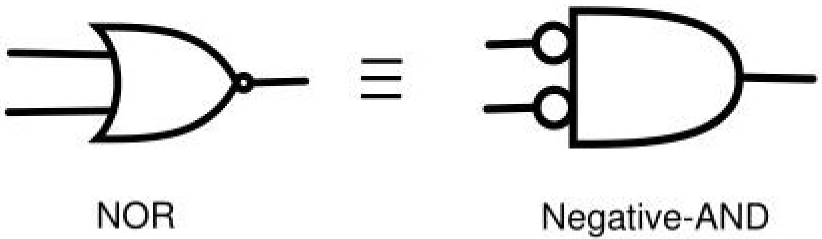
- First submission
  - You can get between 0% and 100% for HW total
  - Correct problems – full credit, no change on resubmission
  - Incorrect problems – no credit, can be corrected in resubmission
- Second submission
  - Submit corrected work
  - Must include a reflection to change a 0% to 100%
    - Why did you get it wrong the first time and what did you learn to correct the error
    - Full sentences

# Test

- Next Wednesday – 02/23
- In class
- Turn in paper submissions
- All material through this week is fair game
- 1 handwritten cheat sheet, both sides
- Calculator

# NAND and NOR Equivalents




| A | B | A **NAND** B |
| --- | --- | --- |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| A | B | A **NOR** B |
| --- | --- | --- |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

- Foreshadowing…

# DeMorgan’s Theorem

- The mathematical equivalent
- $(XY)' = X' + Y'$
- $(X + Y)' = X'Y'$

# Examples

- $(XYZ)'$
- $(X' + Y' + Z')'$
- $(W'X + Y'Z')'$
- $((A' + B') + C')'$
- $(A'B(C + D') + E)'$

# DeMorgan's and XOR

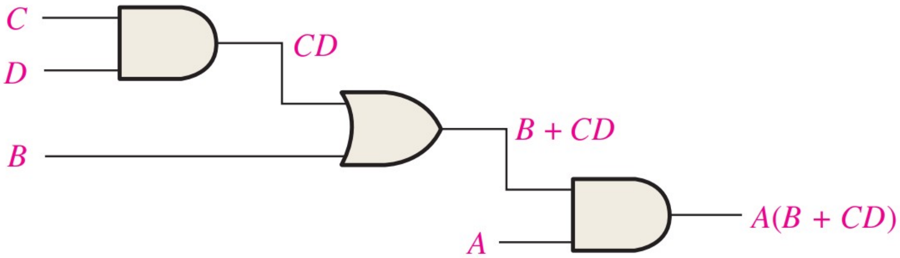
- Substitution required
-  B = AB' + A'B

- Using the Laws and 12 Rules
- XNOR = ( B)' = A'B' + AB

| A | B | A **XOR** B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# Analysis of Circuits

- Start at the left and work right
  - (Beginning to end of circuit)
- Carry a logical output forward as an



# Analysis of Circuits

- Make a truth table
  - 2 ^ #inputs is how many rows there are

| A | B | C | D | CD | B+CD | A(B+CD) |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |

| A | B | C | D | CD | B+CD | A(B+CD) |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

# Simplification

- Boolean Algebra
- Karnaugh Maps

# Boolean Algebra

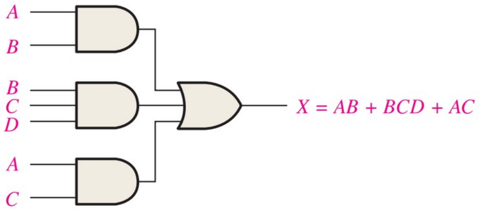
- Simplify expressions using the laws and 12 rules
  - Group terms together that have common terms
  - Simplify negations using DeMorgan's
  - Remove un-needed terms using the rules
    - A + 0 = A, A • 1 = A, etc.

# Examples

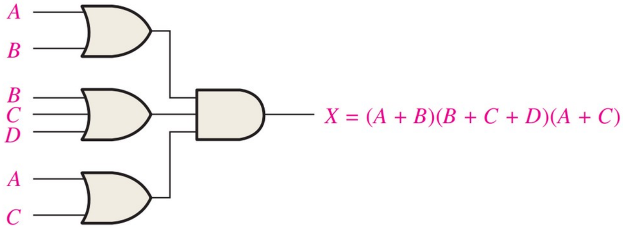
- A’BC + AB’C’ + A’B’C’ + AB’C + ABC
- (AB)’ + (AC)’ + A’B’C’
- A + AB + AB’C

# Standard Forms

- Sum-of-products (SOP)




- Product-of-sums (POS)




- Standard forms makes things easier

# SOP

- All terms have + or • between them and no ()
- AB + AC + ABC ✓
- A'B + AC + A'B'C' ✓
- A(B + C) + AB ✗
- (AB)' + AC ✗

# Making SOP

- Distribute and DeMorgan's
- A(B + C) + AB = AB + AC ✓
- (AB)' + AC = A' + B' + AC ✓

# Standard SOP

- All inputs need to be in each product term

- A’BC + AB’C + A’B’C ✓
- ABC + AB + A’BC’ ✗

# Expanding SOP to Standard SOP

- Use Rule 6: A + A’ = 1
- Multiply terms by the missing variable using Rule 6 and expand
- A’BC + AB + A’BC’
- AB(C + C’) = ABC + ABC’
- A’BC + ABC + ABC’ + A’BC’

# Why?

- Each term in the final expression is a line in the truth table for a 1
- You could read the truth table from the expression
- A’BC + ABC + ABC’ + A’BC’ = 0’11 + 111 + 110’ + 0’10’

# Example

- AB + AB’CD
- Expand twice

# POS

- All terms are groupings of +, multiplied together
- Not can only apply to one input term at a time
- (A’ + B)(A + B + C’)(A) ✓
- (A + B)’(AB + C) ✗

# Making POS

- Use DeMorgan’s and Rule 12: (A + B)(A + C) = A + BC
- (A + B)’ = A’B’ = (A’)(B’) ✓
- (AB + C) = (A + C)(B + C) ✓

# Standard POS

- All inputs need to be in each sum term

- (A’ + B + C)(A + B’ + C)(A’ + B’ + C) ✓
- (A + B + C)(A + B)(A’ + B + C’) ✗

# Expanding POS to Standard POS

- Use Rule 8: $A \bullet A' = 0$ to add terms
- Expand using Rule 12
- $(A' + B + C)(A + B)(A' + B + C')$
- $(A + B + CC') = (A + B + C)(A + B + C')$
- $(A' + B + C)\ (A + B + C)(A + B + C')(A' + B + C')$

# Why?

- Each term in the final expression is a line in the truth table for a 0
- You could read the truth table from the expression
- (A’ + B + C) (A + B + C)(A + B + C’)(A’ + B + C’) = (1’+0+0)(0+0+0)(0+0+1’)(1’+0+1’)

# Example

- $(A + B)(A + B' + C + D)$
- Expand twice

# Reading

- This lecture
  - Sections 4.3-4.6
- Next lecture
  - Sections 4.7-4.10