denumerable + computability

Thursday, December 5, 2024 12:05 PM

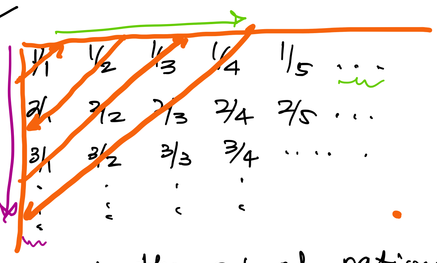
goal: show that ∃ some languages that are not computable.

proof:
1. show number of turing machines is denumerable. ✓
2. show number of languages is not denumerable
3. ⇒ ∃ more languages than tms.
4. turing's strong thesis ⇒ anything that is computable can be computed by a tm.
5. ∴ ∃ languages that are not computable

---

1. show set of all tms is denumerable

of



⇒ the set of rational is denumerable

is the set of all tms denumerable?
yes. convert the tms into a binary string, order them by value, + count.

2. show the set of all languages is not denumerable.

∴ reals are not denumerable. [0,1]

proof by contradiction.
Assume reals ([0,1]) are denumerable.

N	R
1	0.13579...
2	0.25863...
⋮	⋮

r_{new} = 0.26...

⇒ ∃ r_{new} ∉ comprehensive list.

∃ r_{newnew}.

∴ reals are not denumerable.

S = {1, 2, 3} P(S) = {{}, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}, ∅}

- S_d = {a, b, aa, ab, ba, bb, aaa, ...}

P(S_d) = {{a}, {b}, {aa}, ..., {a, aa}, {a, ab, ba}, {a, aa, aaa, aaaa, ...}, }

P(S_d) = {10000..., 01000..., 001000..., 1001100..., }

is the powerset of a denumerable set, denumerable - no

Assume is denumerable

N	P
1	1000...
2	0100...
3	0010000...
4	1100...

p_{new} = 0001

p_{new} ∉ list

∴ assumption incorrect and power set of denumerable is not denumerable.