Chapter 10 extra problems

P 10.3-9 For the circuit shown in Figure P 10.3-9, find (a) the impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in polar form, (b) the total combined impedance in polar form, and (c) the steady-state current $i(t)$.

Answer:

(a) $\mathbf{Z}_1 = 5\angle 53.1°;\ \mathbf{Z}_2 = 8\sqrt{2}\angle -45°$

(b) $\mathbf{Z}_1 + \mathbf{Z}_2 = 11.7\angle -20°$

(c) $i(t) = (8.55)\cos(1250t + 20°)$ A

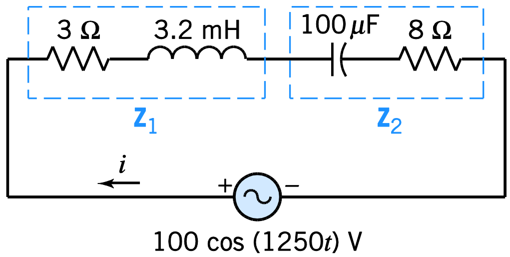
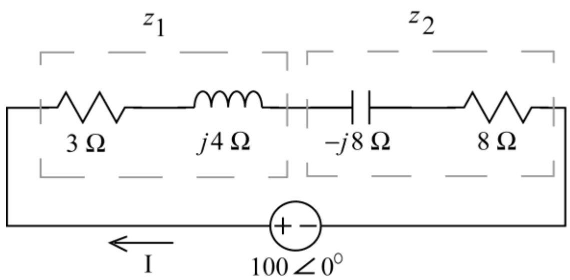


Figure P 10.3-9

Solution:



(a) $\underline{\mathbf{Z}_1 = 3 + j4 = 5\angle 53.1°\ \Omega}$ and $\underline{\mathbf{Z}_2 = 8 - j8 = 8\sqrt{2}\ \angle -45°\ \Omega}$

(b) Total impedance $= \mathbf{Z}_1 + \mathbf{Z}_2 = 3 + j4 + 8 - j8 = 11 - j4 = \underline{11.7\angle -20.0°\ \Omega}$

(c) $\mathbf{I} = \dfrac{100\angle 0°}{\mathbf{Z}_1 + \mathbf{Z}_2} = \dfrac{100}{11.7\ \angle -20°} = \dfrac{100}{11.7}\angle 20.0° \Rightarrow \underline{i(t) = 8.55\ \cos\ (1250t + 20.0°)\ \text{A}}$

P 10.3-10 The circuit shown in Figure P 10.3-10 is at steady state. The voltages $v_s(t)$ and $v_2(t)$ are given by

$$v_s(t) = 7.68 \cos(2t + 47°) \text{ V}$$

and

$$v_2(t) = 1.59 \cos(2t + 125°) \text{ V}$$

Find the steady-state voltage $v_1(t)$.

Answer: $v_1(t) = 7.51 \cos(2t + 35°)$ V

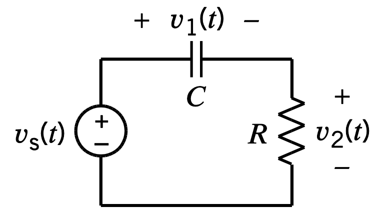


Figure P 10.3-10

Solution:

$$\begin{aligned}\mathbf{V}_1(\omega) = \mathbf{V}_s(\omega) - \mathbf{V}_2(\omega) &= 7.68\angle 47° - 1.59\angle 125° \\ &= (5.23 + j5.62) - (-0.91 + 1.30) \\ &= (5.23 + 0.91) + j(5.62 - 1.30) \\ &= 6.14 + j4.32 \\ &= 7.51\angle 35°\end{aligned}$$

$$v_1(t) = 7.51 \cos(2t + 35°) \text{ V}$$

P 10.3-11 The circuit shown in Figure P 10.3-11 is at steady state. The currents $i_1(t)$ and $i_2(t)$ are given by

$$i_1(t) = 744 \cos(2t - 118°) \text{ mA}$$

and

$$i_2(t) = 540.5 \cos(2t + 100°) \text{ mA}$$

Find the steady-state current $i(t)$.

Answer: $i(t) = 460 \cos(2t + 196°)$ mA

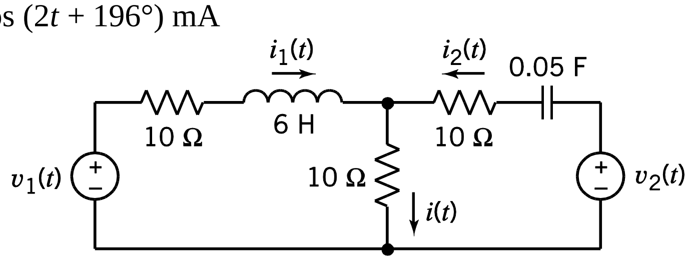


Figure P 10.3-11

Solution:

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.744\angle -118° + 0.5405\angle 100 &= (-0.349 - j\,0.657) + (-0.094 + j0.532) \\ &= (-0.349 - 0.094) + j(-0.657 + 0.532) \\ &= -0.443 - j\,0.125 \\ &= 0.460\angle 196°\end{aligned}$$

$$i(t) = 460\cos(2t + 196°)\ \text{mA}$$

P 10.3-12 Determine $i(t)$ of the *RLC* circuit shown in Figure P 10.3-12 when

$$v_s = 2\cos(4t + 30°)\ \text{V}.$$

Answer: $i(t) = 0.185\cos(4t - 26.3°)$ A

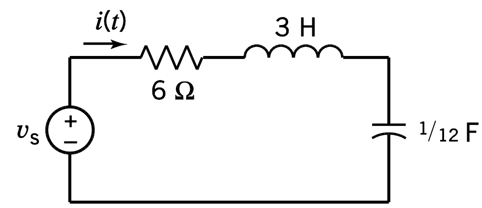
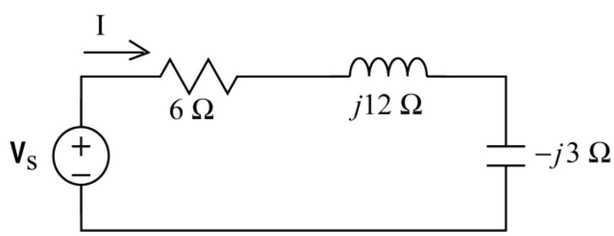


Figure P 10.3-12

Solution:

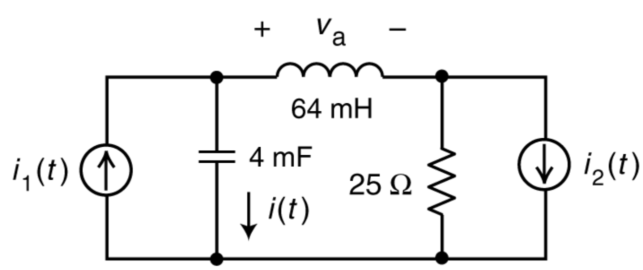


$$\mathbf{V}_s = 2\angle 30°\ \text{V}$$

and $$\mathbf{I} = \frac{2\angle 30°}{6 + j12 + 3/j} = 0.185\angle -26.3°\ \text{A}$$

$$\underline{i(t) = 0.185\cos(4t - 26.3°)\ \text{A}}$$

P10.4-3 Represent the circuit shown in Figure P10.4-3 in the frequency domain using impedances and phasors.

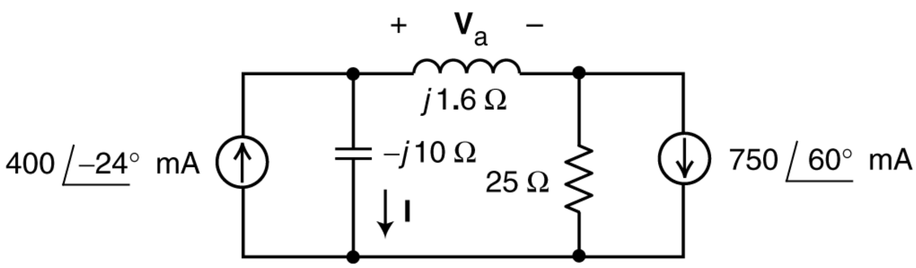


$$i_1(t) = 400\cos(25t - 24°)\ \text{mA}$$

$$i_2(t) = 750\cos(25t + 60°)\ \text{mA}$$

Figure P10.4-3

Solution:



P10.4-7 The input to the circuit shown in Figure P10.4-7 is the current

$$i(t) = 82\cos(10000t)\ \mu A$$

Determine the voltage, *v*(*t*), across the 50 kΩ resistor.

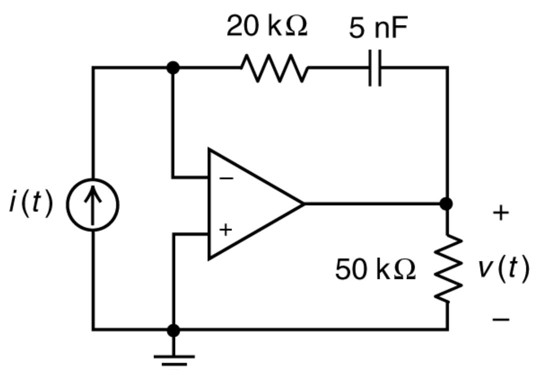
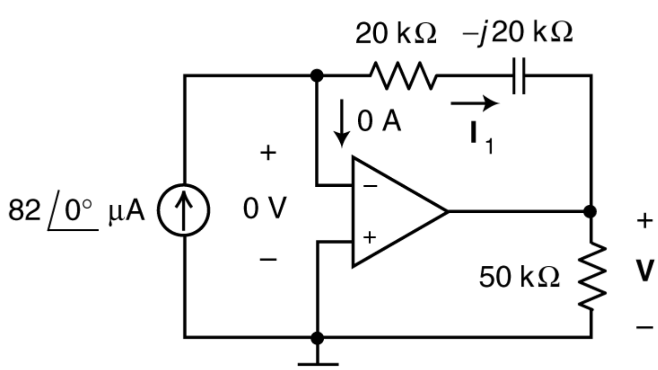


Figure P10.4-7

Solution: Represent the circuit in the frequency domain using phasors and impedances:



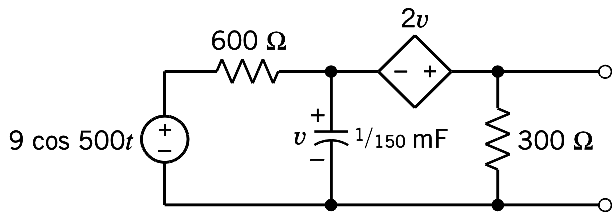
Using KCL $$82\times10^{-6}\angle0° = 0 + \mathbf{I}_1$$

Using KVL $\left(20\times10^{3}\right)\mathbf{I}_1+\left(-j\,20\times10^{3}\right)\mathbf{I}_1+\mathbf{V}=0$

$$\mathbf{V}=-\left(20\times10^{3}-j\,20\times10^{3}\right)\left(82\times10^{-6}\right)=2.3193\angle135°\ \text{V}$$

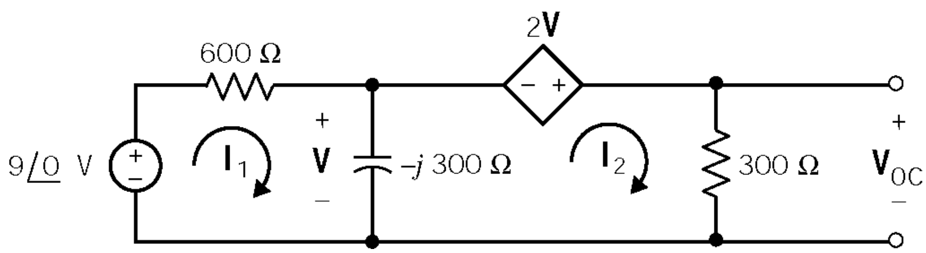
In the time domain $v(t)=2.3193\cos\left(10000\,t+135°\right)\ \text{V}$

P10.7-3 Determine the Thevenin equivalent of this circuit



Solution:

First, determine $\mathbf{V}_{oc}$:



The mesh equations are

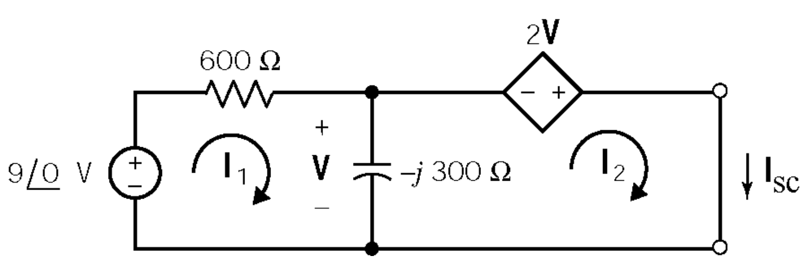
$$600\,\mathbf{I}_1-j300\,(\mathbf{I}_1-\mathbf{I}_2)=9\quad\Rightarrow\quad(600-j300)\,\mathbf{I}_1+j300\,\mathbf{I}_2=9\angle0°$$

$$-2\,\mathbf{V}+300\,\mathbf{I}_2-j300\,(\mathbf{I}_1-\mathbf{I}_2)\ =\ 0\quad\text{and}\quad\mathbf{V}=j300\,(\mathbf{I}_1-\mathbf{I}_2)\quad\Rightarrow\quad j3\,\mathbf{I}_1+(1-j3)\,\mathbf{I}_2=0$$

Using Cramer's rule: $\mathbf{I}_2=0.0124\angle-16°\ \text{A}$

Then $\mathbf{V}_{oc}=300\,\mathbf{I}_2=3.71\angle-16°\ \text{V}$

Next, determine $\mathbf{I}_{sc}$:

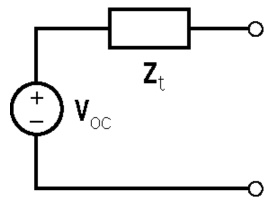


$$-2\,\mathbf{V}-\mathbf{V}=0\quad\Rightarrow\quad\mathbf{V}=0\quad\Rightarrow\quad\mathbf{I}_{sc}=\frac{9\angle0°}{600}=0.015\angle0°\ \text{A}$$

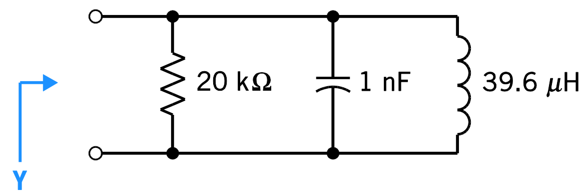
The Thevenin impedance is

$$\mathbf{Z}_T = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{3.545\angle{-16°}}{0.015\angle 0°} = 247\angle{-16°}\ \Omega$$

The Thevenin equivalent is



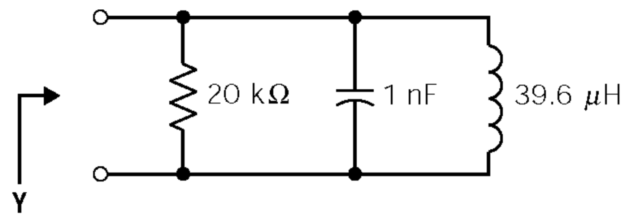
P10.7-5 Determine the frequency at which Y is a pure conductance



Solution:

$$\mathbf{Y} = G + \mathbf{Y}_L + \mathbf{Y}_C$$

$$\mathbf{Y} = G \text{ when } \mathbf{Y}_L + \mathbf{Y}_C = 0 \text{ or } \frac{1}{j\omega L} + j\omega C = 0$$

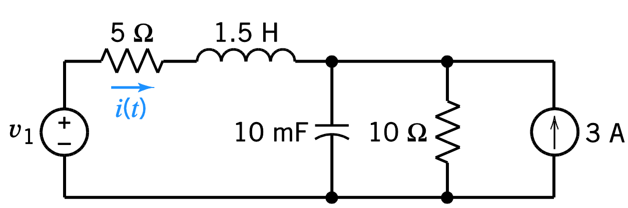


$$\omega_O = \frac{1}{\sqrt{LC}},\ f_O = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{39.6\times 10^{-15}}}$$

$$= 0.07998\times 10^7\ \text{Hz}=800\ \text{kHz}$$

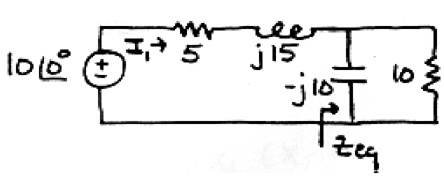
(80 on the dial of the radio)

P10.8-9 Determine the current $i(t)$ for this circuit when $v_1(t)$=10cos(10t) V



Solution:

Use superposition. First, find the response to the voltage source acting alone:



$$\mathbf{Z}_{eq} = \frac{-j10\cdot 10}{10-j10} = 5(1-j)\ \Omega$$

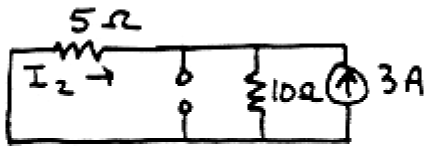
Replacing the parallel elements by the equivalent impedance. The write a mesh equation :

$$-10+5\,\mathbf{I}_1+j15\,\mathbf{I}_1+5(1-j)\,\mathbf{I}_1=0 \quad \Rightarrow \quad \mathbf{I}_1 = \frac{10}{10+j10} = 0.707\angle-45°\text{ A}$$

Therefore:

$$i_1(t) = 0.707\cos(10t-45^{\circ})\text{ A}$$

Next, find the response to the dc current source acting alone:



Current division: $I_2=-\dfrac{10}{15}\times 3 = -2\text{ A}$

Using superposition:

$$i(t) = 0.707\cos(10t-45°)-2\text{ A}$$

P10.10-1 The input to the circuit shown in Figure P10.10-1 is the voltage

$$v_s(t)=2.4\cos(500t)\text{ V}.$$

Determine the output voltage $v_o(t)$.

Answer: $v_o(t)=6.788\cos(500t+135°)\text{ V}$

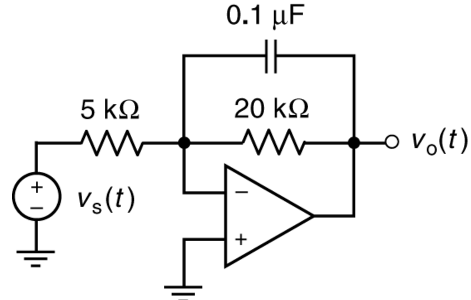
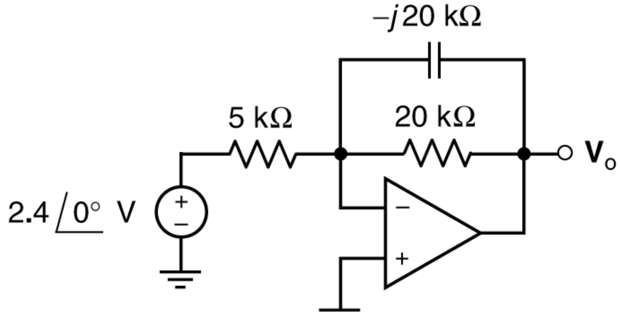


Figure P10.10-1

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as an inverting amplifier, we can write

$$\mathbf{V}_{\text{o}} = \left(-\frac{20 \parallel -j\,20}{5}\right)(2.4\angle 0) = \left((1\angle 180°)\frac{14.14\angle -45°}{5}\right)(2.4\angle 0) = 6.788\angle 135° \text{ V}$$

In the time domain $$v_{\text{o}}(t) = 6.788\cos(500\,t + 135°) \text{ V}$$

(Checked using LNAPAC 3/15/12)

P10.10-2 The input of the circuit shown in Figure P10.10-2 is the voltage

$$v_s(t) = 1.2\cos(400t + 20°)\ \text{V}.$$

Determine the output voltage $v_o(t)$.

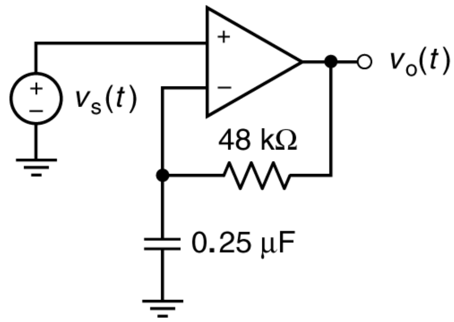
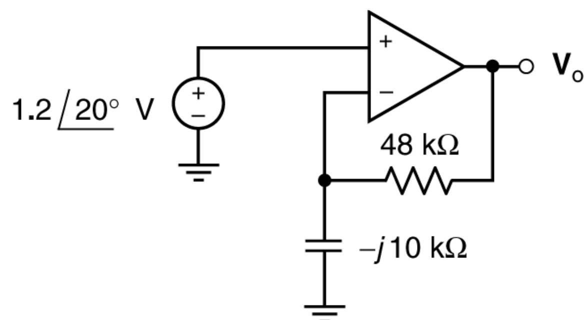


Figure P10.10-2

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as a noninverting amplifier, we can write

$$\mathbf{V}_0 = \left(1 + \frac{48}{-j10}\right)(1.2\angle 20°) = (1 + j\,4.8)(1.2\angle 20°) = 5.88\angle 98°\ \text{V}$$

In the time domain $$v_o(t) = 5.88\cos(400t + 98°)\ \text{V}$$